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Stability Analysis of Thin-Walled Members with Curved Cross-Section Parts: Inelastic Behavior

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Stability analysis of thin-walled members with curved cross-section parts: inelastic behavior

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Abstract

In this paper the buckling behavior of thin-walled members with cross-sections with curved parts is investigated. Due to the curved parts, shell-like buckling is a potential mode of failure. The objective of the research is to understand whether shell-like buckling behavior might be governing in practical cold-formed steel members. For this aim, numerical studies have been carried out, involving linear buckling analysis as well as nonlinear analysis with imperfections, by considering various cross-sections. Based on the results it is concluded that shell-like behavior might be critical in certain cases.

Introduction

As linear cold-formed steel profiles have become everyday solutions in many applications (e.g., purlins, rafters), several research activities started with aiming to develop more efficient cross-sections. These research and/or innovation activities led to more refined cross-section shapes, e.g. with multiple longitudinal stiffeners. Lately, attempts for a more formal mathematical optimization have been reported by various research groups, see e.g., Gilbert et al. (2012), Leng et al, (2014), Moharrami et al. (2014). In many cases the found optimal cross-section shapes tend to consist of curved parts rather than flat parts, at least if no special constraints are used to avoid the formation of curved parts.

Though the highly curved cross-section shapes might be impractical, it is reasonable to assume that some combination of flat and curved parts might be feasible and advantageous, e.g., by assuming some classical cross-section, but with unusually large corner radii. The problem is, however, that the behavior of such thin-walled members with curved cross-section parts is not yet investigated in a comprehensive manner, therefore it is questionable whether the reported optimal cross-sections are properly analyzed by considering all possible failure modes. Namely: since curved cross-section parts mean cylindrical surfaces, shell-like behavior is theoretically possible, but shell-like behavior is not

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considered in current cold-formed steel member design. Note, though plate-like and shell-like buckling are geometrically similar, both being associated with small buckling waves, they might have significantly different post-buckling behavior: plate-like buckling has typically considerable post-buckling reserve (i..e, the load-bearing capacity might be considerably above the critical load), in case of shell-like behavior, however, the capacity is typically much smaller than the critical load. Therefore, proper distinction in between plate-like and shell-like buckling can (and will) be made based on the post-buckling behavior.

In this paper the results of numerical parametric studies are presented. The calculations are completed by shell finite element analysis. Both column and beam members are investigated, considering two cross-section topologies, but a large number of curved and non-curved cross-sections, by systematically changing the corner radii in a wide range. In this paper linear buckling analysis, and geometrically and materially nonlinear analysis with imperfections (i.e., GMNI analysis) are presented. The results suggest that in certain cases shell-like behavior should be considered in predicting the capacity.

Overview, solution strategy

The objective of the research is to check whether shell-like buckling can or cannot be governing in case of thin-walled cold-formed steel column and beam members. In other words, we want to check whether the presence of curved parts in the cross-section geometry deteriorates the post-buckling reserve of the buckling (i.e., buckling characterized by small waves). The aim is not to investigate specific products, but to analyse the phenomena. Therefore, only simple cross-section geometries are selected. One single cross-section topology is chosen for pure compression, and another one for pure bending. The topology for compression is a doubly-symmetrical hollow section shape, (with a maximum dimension of 100 mm,) while the topology for bending is a C-like singly-symmetrical open cross-section shape (with 100 mm width and 130 mm height). (Note, this slightly unusual lipped-channel geometry is selected in order to make eliminate distortional buckling and/or buckling of the lip.) Since the emphasis is on the curved parts, within the given topology the corners are rounded with variable corner radius, the radius being varying in between zero (i.e., sharp corners) and the physically possible maximum (i.e., 50 mm). In case of the hollow section, therefore, the increasing radius transforms the shape from a square hollow section (SHS) to a circular hollow section (CHS), as shown in Fig. 1. The figure shows the considered C-like shapes with the changing corner radius, too.

Since the aim here is to analyze buckling with short buckling waves, only short members are considered, with a length equal to 200 or 300 mm, which is roughly twice as much as the maximum cross-section dimension for the SHS type and the C-like section respectively. The selection of short member length automatically eliminates the global buckling phenomena. It is also to mention that distortional buckling is practically also eliminated by the selection of member length and cross-section shapes. In case of hollow sections distortional buckling mode theoretically exists, however, the associated critical load is much larger than those belong to local-plate buckling, hence, it is reasonable to assume that the effect of distortional buckling for the considered column problems is negligible. In case of C-like cross-sections distortional buckling is typically important, however, in our cases the flange lips are relatively large, and if such a cross-section is subject to bending, the lips are lightly compressed, hence distortional buckling and/or lip buckling has minor role.

The final goal of the numerical studies is to estimate the load-bearing capacity of the members with (and without) significant curved parts. In the lack of real experiments, the load bearing estimation is carried out by finite element analysis, using shell finite elements, and considering material and geometric nonlinearity with imperfections (i.e. GMNI analysis). Only geometric imperfections are used, taken as properly scaled buckling shapes.

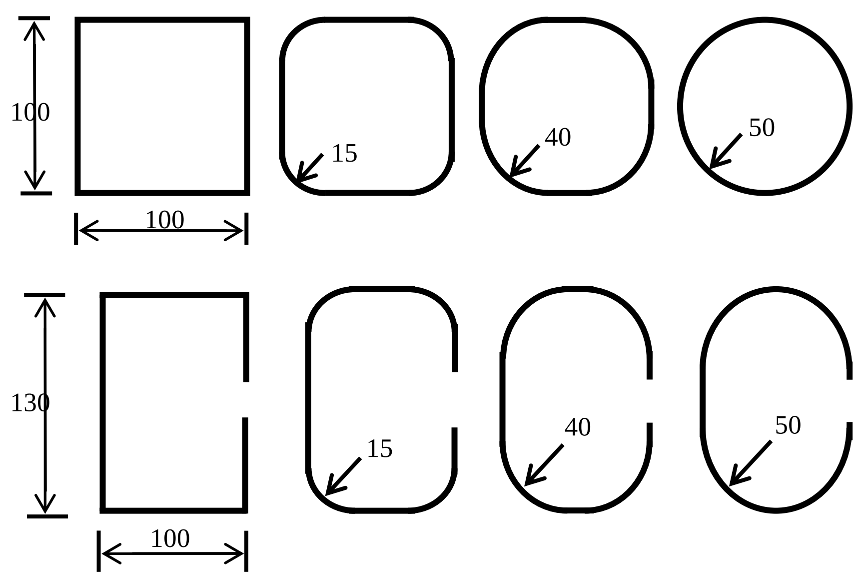


Figure 1: Cross-sections

The major steps of the research work therefore are as follows:

- parametric finite element model definition,
- linear buckling analysis for a large number of cases, by systematically changing the model parameters in a wide range,
- development of a method to numerically characterize the buckling modes (in an automated way),
- imperfection sensitivity analyses by using elastic material and geometric nonlinear analysis with geometric imperfections (i.e., GNI analysis),
- load bearing capacity estimation with geometrically and materially nonlinear analysis (i.e., GMNI analysis). .

Based on the results of the GMNI analyses the load bearing capacity of the members can be assessed and conclusion can be drawn.

In this paper the focus is on the GMNI analysis, while GNI analysis is discussed in Ádány et al (2016).

Finite Element model

For the parametric studies a parametric finite element model was built in Ansys. The geometry of the analyzed cross-section topologies is illustrated in Fig. 1. Eight-node quadratic shell element have been used, with six degrees of freedom at each node. This element is called SHELL281 in Ansys terminology. A relatively fine mesh is used, the total degrees of freedom being approx. 34000-47000 in case of the SHS-like sections and 51000-98000 in the C-like sections. The size of the equation system was a key factor since thousands of cases have been investigated, therefore, a balance had to be kept in between accuracy and running time. It is to mention that some other element types have been tested, too, but it was concluded that there is no significant difference in the results if appropriate mesh density is chosen.

A globally and locally hinged support was defined for both end sections. Warping is restrained. One may think of this support arrangement as if thick plates were welded to the end cross-sections, and the plate is supported in one point by a hinge (i.e. by restraining translations and twisting rotation around the longitudinal axis of the member, while allowing the rotations around the other axes). Practically, a master node is defined at each end to which each end cross-section node is linked by rigid constraint equations. It is to note that some slightly different support arrangements were also considered, but it had not any significant influence on the local behavior.

Linear buckling analysis

Linear buckling analyses are performed for both cross-section topologies, with varying corner radius and thickness. More specifically, the thickness varied from 0.4 mm to 1.0 mm by 0.1 mm steps and from 1.0 mm to 3.0 mm by 0.2 mm steps. while the corner radius varied from zero to the physically possible maximum 50 mm by 5 mm steps. Altogether 685 cases are analyzed, and in each case the first 200-300 critical loads and corresponding buckled shapes are calculated. (Note, in certain cases much more modes are calculated, up to 2-3000 modes.) Some of the modes are shown in Figs. 2 and 3.

In general, if the deformations are concentrated to the flat parts of the member (while the curved parts are subject to much smaller deformations), the buckled shape is most likely "plate-like" buckling. On the other hand, if significant deformations appear at the curved parts, the buckled shape is most likely "shell-like" buckling. If deformations are important in both the flat and curved parts, the mode is considered as "mixed".

By the visual inspection of the buckling modes it can be concluded that:

- in case of small corner radius (r<25 mm) the first few hundred buckling modes can be classified as (classic) plate-like modes,
- in case of larger corner radius the first buckling modes are plate-like, but shell-like modes appear among the higher modes,
- the larger the corner radius, the sooner the shell-like buckling appears,
- both "shell-type" (see Fig. 2, #110) and "axisymmetric-type" (see Fig. 2, #158) modes appear, however, axisymmetric modes are found only as very high modes and/or in case of very large corner radius,
- the increasing tendency of the critical loads are dependent on the cross-section shape: the larger the corner radius, the slower the increasing of the critical loads (e.g., in case of a hollow section with r=5 mm and t=1 mm, the ratio of the 200th to the 1st critical load is 16.8, while the same ratio is 4.3 if the radius is 40 mm).

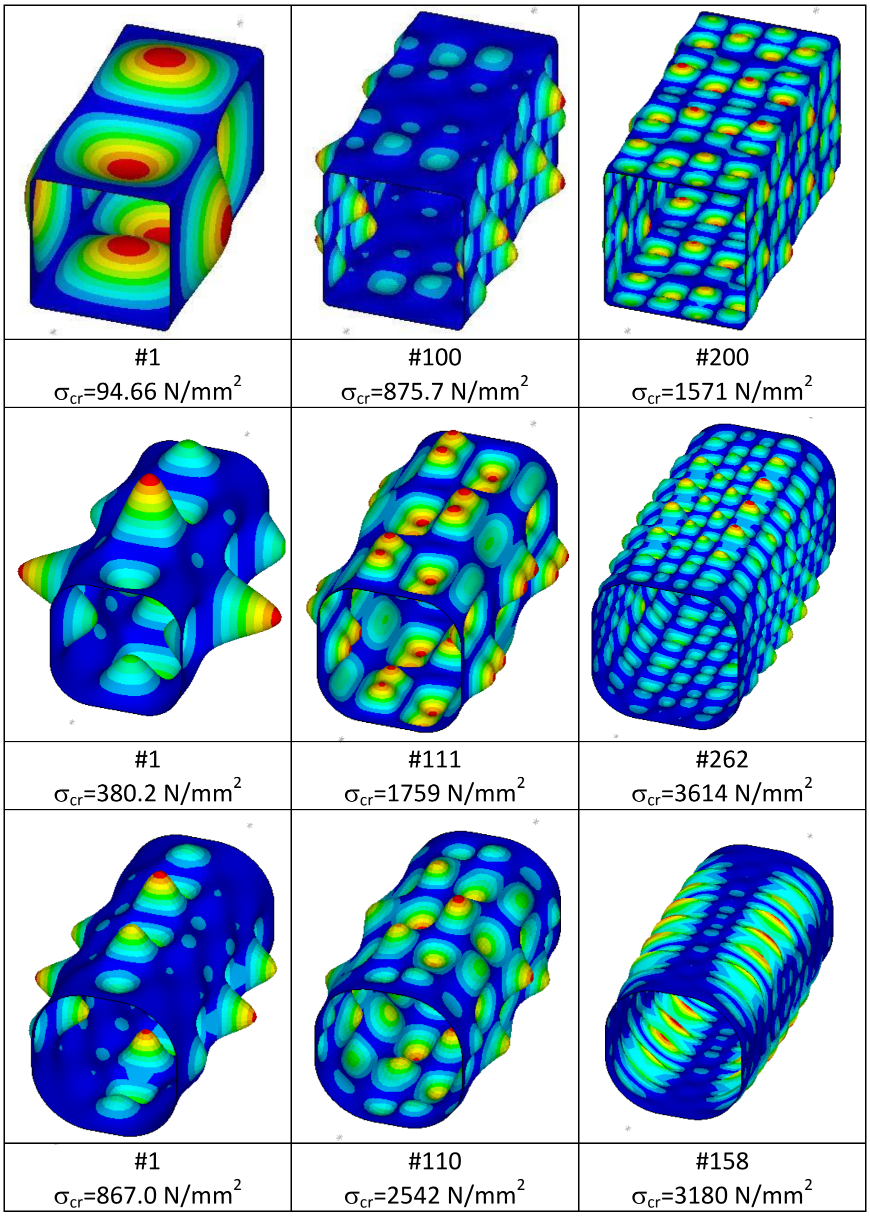
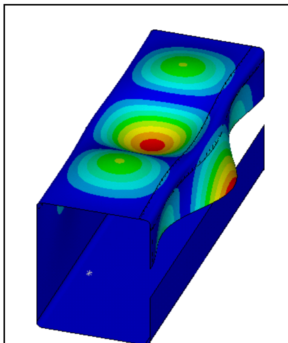
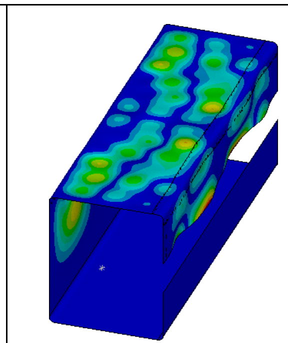
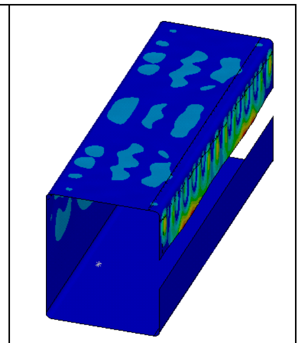


Figure 2: Buckling shapes of SHS-like sections, r=5-30-40 mm, t=1.0 mm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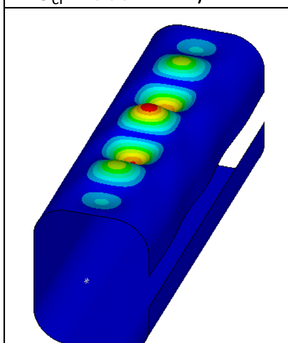	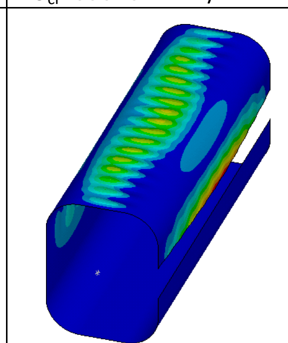	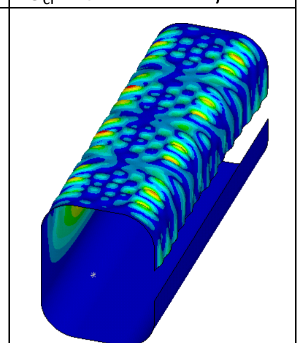
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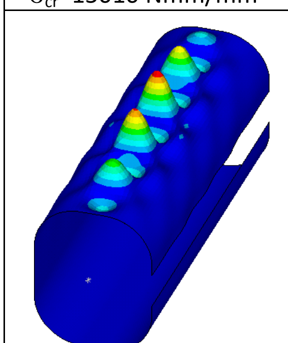	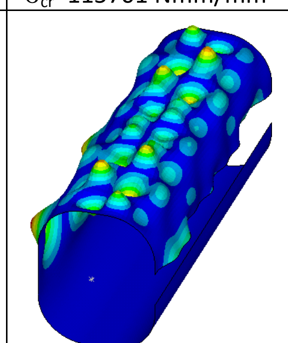	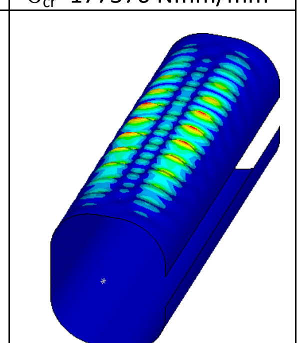
#1 σ_{cr}=32103 Nmm/mm^2	#67 σ_{cr}=99203 Nmm/mm^2	#89 σ_{cr}=120554 Nmm/mm^2

Figure 3: Buckling shapes of C-like sections, *r*=5-30-40 mm, *t*=1.0 mm

Spectral analysis of the buckling shapes

Buckled shapes are intended to be used as geometric imperfections in nonlinear analyses for a large number of cases. It is known that the various buckling modes have very different post-critical behavior. It is expected, therefore, that the member will show significantly different imperfection sensitivity depending on the nature of the imperfection, i.e., depending on the nature of the buckling mode which is used as geometric imperfection. Since we have many different cross-section shapes, and hundreds of buckling modes for each case, it is highly beneficial to be able to numerically characterize the buckled shapes, which numerical characterization might later be connected to the imperfection sensitivity (or: post-critical behavior).

Here a simple and automatic characterization is proposed and used, which can be summarized as follows:

- longitudinal straight lines are defined at some characteristic points of the member,
- the displacements along the lines are collected,
- the displacement function along each line is approximated by trigonometric series,
- the coefficients of interpolation functions are normalized.

Since in most of the cases only a few coefficients have non-zero values, the few non-zero coefficients show the characteristic buckling length(s), as well as highlight those parts of the member where the deformations are dominant.

To illustrate the spectral analysis of the buckling shapes, a hollow section with *r*=40mm and *t*=1mm is considered here, with the 3 buckling modes shown in Fig. 2. The straight lines are defined as shown in Fig. 4, namely: two in the flat part of the cross-section (f1,f2), and two in the curved part (c1,c2). Table 1 shows the normalized coefficients for the 4 lines.

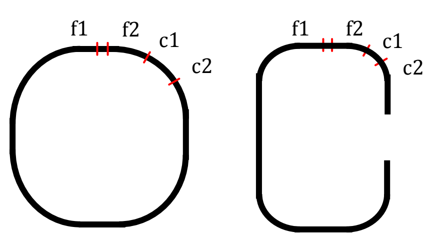

Figure 4: Position of longitudinal lines for spectral analysis of buckling shapes

Table 1. Spectral decomposition of selected buckling modes

Hollow section *r*=40mm, *t*=1mm												
Buckl. shape #1				Buckl. shape #110				Buckl. shape #158				Half-wave length mm
Node sets				Node sets				Node sets				
f1	f2	c1	c2	f1	f2	c1	c2	f1	f2	c1	c2	
0	0	0	0	0	4	1	1	0	0	0	0	200
1	1	0	0	0	0	0	0	1	1	0	1	100
0	0	0	0	0	42	18	19	0	0	0	0	67
3	3	0	0	0	0	0	0	1	0	3	0	50
0	0	0	0	0	100	32	38	0	0	0	0	40
100	85	2	2	0	0	0	0	0	0	1	0	33
0	0	0	0	0	16	1	6	0	0	0	0	29
71	61	1	1	0	0	0	0	0	0	1	0	25
0	0	0	0	0	9	1	2	0	0	0	0	22
0	0	0	0	0	0	0	0	0	0	1	1	20
0	0	0	0	0	7	1	1	0	0	0	0	18
0	0	0	0	0	0	0	0	0	0	1	3	17
0	0	0	0	0	7	0	0	0	0	0	0	15
0	0	0	0	0	0	0	0	0	0	4	7	14
0	0	0	0	0	20	1	1	0	0	0	0	13
0	0	0	0	0	0	0	0	2	1	17	61	12.5
0	0	0	0	0	6	0	0	0	0	0	0	12
0	0	0	0	0	0	0	0	4	2	37	100	11
0	0	0	0	0	1	0	0	0	0	0	0	10.5
0	0	0	0	0	0	0	0	1	0	2	14	10

It can be observed that

- mode #1 is clearly plate-like buckling since the numbers (and thus the deformations) in the corner region (c1,c2) are much smaller than those in the flat parts (f1,f2),
- mode #158 is clearly shell-like buckling,
- mode #110 shows both plate-like and shell-like characteristics.

It can be concluded, therefore, that the here-introduced spectral analysis of the buckled shapes is simple-to-use, practically automatic (for the considered cases), and makes it possible to geometrically categorize the buckling modes.

Capacity estimation by GMNI analysis

To estimate the load-bearing capacity of the members, geometrically and materially nonlinear analyses are carried out. (Note, since the members are short, and global and distortional behaviour are practically excluded, the calculated load-bearing capacity characterizes the local behaviour only.) Since it is known that different imperfection patterns lead to different nominal capacities, parametric study is performed here by considering a large number of possible imperfection patterns. In all the cases, the imperfection pattern is assumed to be in the shape of that of a linear buckling mode.

The parametric study has been intended to be comprehensive, at least for the selected two cross-section topologies. The varying parameters are the following: the thickness, the corner radius, the imperfection pattern, the imperfection amplitude, and the yield strength of the material. It is realized, however, that a comprehensive parametric study would require unrealistic computation time, therefore, the parameters are carefully selected, as follows.

Based on some preliminary calculations it was concluded that the yield strength does not affect the tendencies (though the numerical values are obviously affected), thus, it was decided to use one single yield strength value, namely: 350 MPa (which is a frequently used basic yield strength for cold-formed steel members).

For the other parameters: we have considered both cross-section topologies, three thickness values: 0.5, 1.0, and 2.5 mm, and (in most of the cases) five corner radius values: 5, 15, 25, 30, and 40 mm.

The number of imperfection patterns may practically be infinite. To have a realistic amount of imperfection patterns, we have selected the first 50-200 linear buckling modes for all the considered cases, plus we have selected the shell-like and mixed modes (by applying the above-described spectral analysis procedure) from the first few thousand buckling modes. This selection of imperfection patterns is based on the observation that the first buckling modes are mostly (or exclusively) plate-like modes and the first dozens of plate-like modes will always contain the most unfavorable plate-like imperfection pattern, therefore, it is enough to select only the shell-like patterns from the higher modes. Thus, this selection of imperfection patterns ensures that all the potentially most unfavorable patterns will be considered, while the number of considered imperfection patterns remains practically acceptable.

As far as imperfection magnitude is concerned, it is known that the general tendency is: the larger the imperfection magnitude is, the smaller the calculated capacity is. However, in many cases the influence of the imperfection magnitude on the calculated capacity is not too significant, at least in the practically important range of possible imperfections. Therefore, our aim was to select a limited number of imperfection magnitudes. In case of plate-like buckling behaviour (of sharp-cornered members), the Eurocode for steel plated elements (CEN 2006) gives guidance for the determination of the magnitude of the initial equivalent imperfection. In case of shell-like buckling behaviour, at least in case of compressed cylindrical shells, guidance is given in the Eurocode for steel shells (CEN 2007). In this latter design standard the value of the imperfection magnitude is greatly dependent on the wall thickness, that is why we have selected one single imperfection magnitude for each considered thickness. The selected initial imperfection magnitudes are: 0.5, 0.7, and 1.5 mm for the thickness of 0.5, 1.0, and 2.5 mm respectively. These values can be regarded as upper limits that are proposed or allowed by the referenced design codes. It is to note, although these values are technically correct, sometimes they seem to be slightly unrealistic, since the half-wavelength of higher buckling modes is normally between 5-20 mm (for the considered cases). Still, it is believed that the performed analyses and the results correctly show the behavior and the tendencies. As far as the actual load-bearing capacities are concerned, the here-presented values can be regarded as realistic estimations (most probably: slightly conservative estimations), but not as precise (design) values.

In the GMNI analysis load-displacement curves are established. The nominal capacity is the maximum point of the load-displacement curve. In order to be able to compare the various cross-sections, we have used a normalized version of the capacity, i.e., the maximum normal force or maximum bending moment divided by the cross-sectional area. Samples are shown in Fig. 5.

Calculated nominal capacities are given in Tables 2 and 3. Three capacity values are given for all the considered cases, as follows. The value earmarked as “first mode” means the calculated capacity if the first linear buckling mode is used as geometric imperfection. The value of “first 10 modes” means the minimal capacity of the capacities calculated with the first 10 buckling modes. Finally, “all modes” capacity is the minimal value among all the considered geometric imperfections (including very high linear buckling modes).

Since the first 10 linear buckling modes are always plate-like modes, the value of “first 10 modes” can be regarded as an estimation of the capacity that belongs to plate-like behaviour. On the other hand, the value “all modes” can be regarded as an estimation of the final capacity.

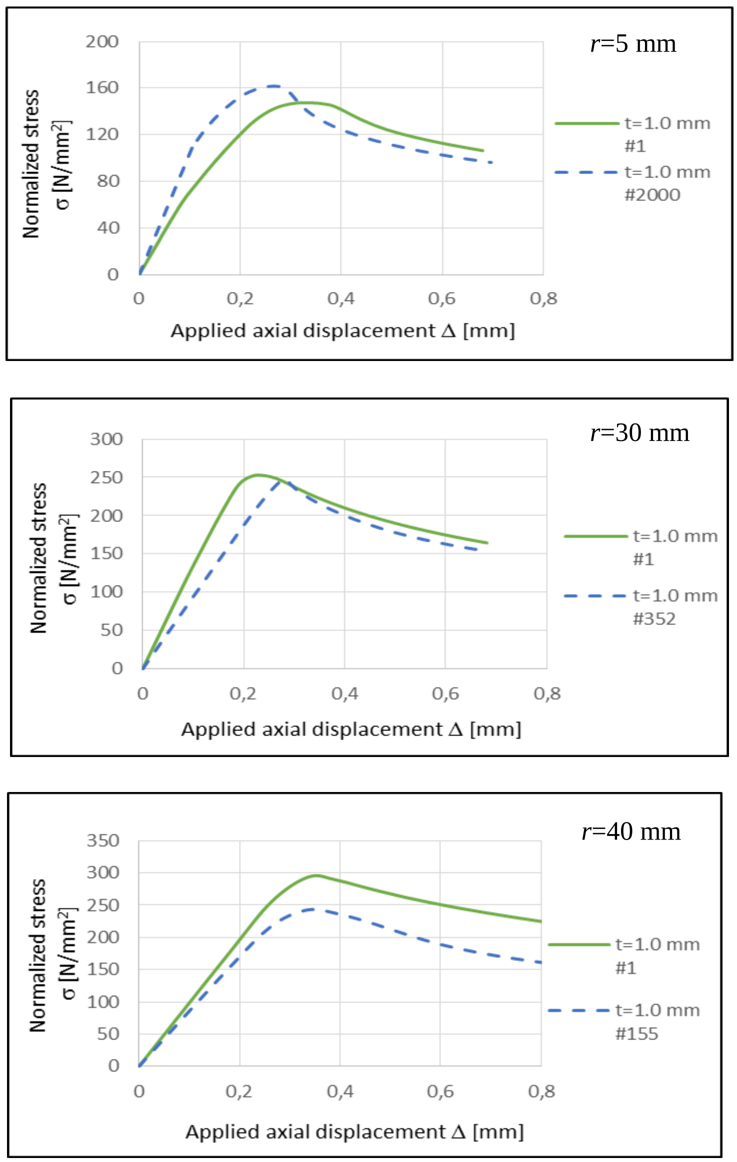

Figure 5: Load-displacement curves from GMNI analysis for SHS-like sections:

Table 2. Estimated capacities for SHS-like sections

		first mode		first 10 modes		all modes	
t mm	*r* mm	capacity N/mm^2	mode nr as imperf	capacity N/mm^2	mode nr as imperf	capacity N/mm^2	mode nr as imperf
0.5	5	88	1	83	10	79	1998
0.5	15	151	1	139	7	122	1106
0.5	25	180	1	180	1	110	603
0.5	30	212	1	204	2	142	472
0.5	40	273	1	257	7	182	218
1.0	5	147	1	147	1	142	19
1.0	15	212	1	190	8	190	8
1.0	25	242	1	237	2	231	435
1.0	30	253	1	253	1	247	352
1.0	40	294	1	292	8	243	155
2.5	5	242	1	242	1	242	1
2.5	15	257	1	257	1	257	1
2.5	25	317	1	289	4	284	283
2.5	30	298	1	297	7	254	139
2.5	40	313	1	308	9	276	74

Table 3. Estimated capacities for C-like sections

		first mode		first 10 modes		all modes	
t mm	*r* mm	capacity Nmm/mm^2	mode nr as imperf	capacity Nmm/mm^2	mode nr as imperf	capacity Nmm/mm^2	mode nr as imperf
0.5	5	5093	1	4511	8	4511	8
0.5	15	7165	1	68515	2	6309	904
0.5	30	9824	1	8946	5	6779	325
0.5	40	11354	1	10156	6	8121	196
1.0	5	8154	1	8154	1	8092	27
1.0	15	9032	1	9032	1	8365	585
1.0	30	12005	1	11972	9	10971	186
1.0	40	12950	1	12950	1	11640	90
2.5	5	13420	1	13417	2	13417	2
2.5	15	13210	1	13210	1	13210	1
2.5	30	14419	1	14370	5	14170	102
2.5	40	15147	1	13894	9	13526	52

As the numerical values of Tables 2 and 3 suggest, the capacity degrading effect of shell-like buckling exists, but only for small thickness and/or large corner radius. It seems that the *r*/*t* ratio must be larger than approx. 20-30 so that the shell-like behavior could become critical. This requires a relatively slender member with unusually large corner radii.

Another important observation is that the calculated capacity increases with the corner radius even in the case of most unfavorable imperfection patterns. Note, however, that in this study only plate-like and shell-like buckling behavior are considered, (while global and distortional buckling are excluded,) therefore the observed beneficial effect of the larger corner radii is interpreted only for the local buckling behavior.

Concluding remarks

In this paper the buckling behavior of thin-walled members with cylindrically curved parts has been investigated. The focus is on the local buckling behavior, including plate-like and shell-like buckling. Parametric numerical studies have been completed on two selected cross-section types, namely hollow section and C-like section, by systematically varying the radius of the curved parts. Elastic linear buckling modes have been determined first, which characterized numerically, then used as geometric imperfections in non-linear analyses. Based on the results the following conclusions can be drawn.

If the curved parts are significant in the cross-section, shell-like buckling is possible. In linear buckling analysis the shell-type modes are among the higher modes. The corresponding critical load values are typically multiples of the lowest critical load value with the tendency as follows: the smaller the cylindrical part of the cross-section, the larger the ratio of the shell-type critical load to the lowest critical load.

By using the buckled shapes as initial geometric imperfections, elastic or inelastic capacities can be calculated. Capacities calculated via a materially and geometrically non-linear analysis with properly scaled geometric imperfections (GMNI) can be regarded as estimations of real capacities. In the actual study only short members and only a few cross-section topologies have been considered, which also means that only local behavior is analyzed. Therefore, the observations are valid only for the capacities that belong to the local behavior, while other behavior modes (e.g. distortional or global buckling) are disregarded.

Based on the results of large number of such GMNI analysis it is concluded that the post-critical behavior of plate-like buckling (i.e., when buckling deformations are mostly at the flat parts of the member) and post-critical behavior of shell-like buckling (i.e., when buckling deformations are concentrated at the curved parts of the member) are distinctly different. It is found that shell-like behavior can be governing for certain cross-section geometries, namely if the radius-to-thickness ratio is larger than approx. 20-30. It is also observed, however, that the unusually large corner radius is beneficial from the local capacity point-of-view, since the general tendency is that the larger the corner radius, the larger the member capacity is. Nevertheless, in case of large corner radius the existing cold-formed steel design procedures must be supplemented to consider shell-like behavior, too.

Acknowledgements

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